Практика 12

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Содержание

$\eta \backslash \xi$	0	2	4	6	q_j
0	0.15	0.1	0.1	0.05	0.4
4	0.1	0.1	0.05	0.1	0.3
8	0.05	0.05	0.1	0.05	0.3
$p+i$	0.3	0.25	0.25	0.2	$\sum = 1$

Задача 1. $p_n = ?$

Задача 2. Частичные ξ и η. $E\xi, D\xi, E\eta, D\eta, \sigma_\xi, \sigma_\eta$

Решение.

ξ_i	0	2	4	6
p_i	0.3	0.25	0.25	0.2

η_i	0	4	8
q_i	0.4	0.3	0.3

$$E\xi = \sum \xi_i p_i = 2.7$$

$$D\xi = 4.91$$

$$\sigma_\xi = 2.216$$

$$E\eta = 3.6$$

$$D\eta = 11.04$$

$$\sigma_e ta = 3.323$$

Задача 3. Независимые?

Решение. $p_{??} = 0.1$
$p_2 \cdot q_2 = 0.25 \cdot 0.3 = 0.075$
Тогда ξ, η независимы

Задача 4.

$$p\left(\frac{\xi^2}{9} + \frac{\eta^2}{25} \leq 1\right) = ?$$

Решение.

$$p\left(\frac{\xi^2}{9} + \frac{\eta^2}{25} = 1\right) - p(\xi = 0, \eta = 0) + p(\xi = 2, \eta = 0) + p(\xi = 0, \eta = 4) = 0.15 + 0.1 + 0.05 = 0.3$$

$$F(x,y) = \begin{cases} 1 - \frac{1}{x^2} - \frac{1}{y^2} + \frac{1}{x^2y^2} & (x,y) \in \{x \geq 1, y \geq 1\} \\ 0 & (x,y) \notin \{x \geq 1, y \geq 1\} \end{cases}$$

Задача 5.

$$f_{\xi,\eta}(x,y) = ?$$

Решение.

$$f_{\xi,\eta} = \frac{\partial^2 F}{\partial x \partial y}$$

$$\frac{\partial F}{\partial x} = \frac{2}{x^2} - \frac{2}{x^3 y^2}$$

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{4}{x^3 y^3}$$

$$f_{\xi,\eta}(x,y) = \begin{cases} \frac{4}{x^3 y^3} & (x,y) \in \{x \geq 1, y \geq 1\} \\ 0 & (x,y) \notin \{\geq 1, y \geq 1\} \end{cases}$$

Задача 6.

$$f_\xi(x) = ? \quad f_\eta(y) = ?$$

Решение.

$$f_\xi(x) = \int_{-\infty}^{+\infty} f_{\xi,\eta}(x,y)\,dy = \int_1^\infty \frac{4}{x^3 y^3}\,dy = \tag{1}$$

$$= -\frac{4}{x^3} \cdot \frac{1}{2y^2}\Big|_1^\infty = -\frac{1}{x^3}\left(\lim_{y\to\infty} \frac{1}{y^2} - 1\right) = \frac{2}{x^3} \tag{2}$$

$$f_\xi(x) = \begin{cases} 0 & x < 1 \\ \frac{2}{x^3} & x \geq 1 \end{cases}$$

$$f_\eta(y) = \begin{cases} 0 & y < 1 \\ \frac{2}{y^2} & y \geq 1 \end{cases}$$

Задача 7.

$$F_\xi(x) = ? \quad F_\eta(y) = ?$$

Решение.

$$F_\xi(x) = \int_{-\infty}^{x} f_\xi(x)\,dx = \int_1^x \frac{2}{x^3}\,dx = -\frac{1}{x^2}\Big|_1^x = 1 - \frac{1}{x^2} \tag{3}$$

$$F_\xi(x) = \begin{cases} 0 & x < 1 \\ 1 - \frac{1}{x^2} & x \geq 1 \end{cases} \tag{4}$$

$$F_\eta(y) = \begin{cases} 0 & y < 1 \\ 1 - \frac{1}{y^2} & y \geq 1 \end{cases} \tag{5}$$

Задача 8. Независимы?

Решение.

$$f_\xi \cdot f_\eta(y) = \frac{2}{x^3}\frac{2}{y^3} = \frac{4}{x^3 y^3} = f_{\xi,\eta}(x,y) \tag{6}$$

$\implies \xi, \eta$ — независимы

$$F_\xi(x) \cdot F_\eta(y) = \left(1 - \frac{1}{x^2}\right)\left(1 - \frac{1}{y^2}\right) = 1 - \frac{1}{x^2} - \frac{1}{y^2} + \frac{1}{x^2 y^2} = F_{\xi,\eta}(x,y) \tag{7}$$

Задача 9.

$$p(2 \leq y < 4, 1 < \eta < 2) = ?$$

Решение.

$$p(2 < \xi < 4, 1 < \eta < 2) = p(2 < \xi < 4) \cdot (1 < \eta < 2) = (F_\xi(4) - F_\xi(2)) \cdot F_\eta(2) =$$

$$= \left(\left(1 - \frac{1}{16}\right) - \left(1 - \frac{1}{4}\right)\right) \cdot \left(1 - \frac{1}{4}\right)$$

$$p(2 < \xi < 4, 1 < \eta < 2) = F(4,2) - F(2,2) - F(4,1) + F(2,1) = \frac{9}{64}$$